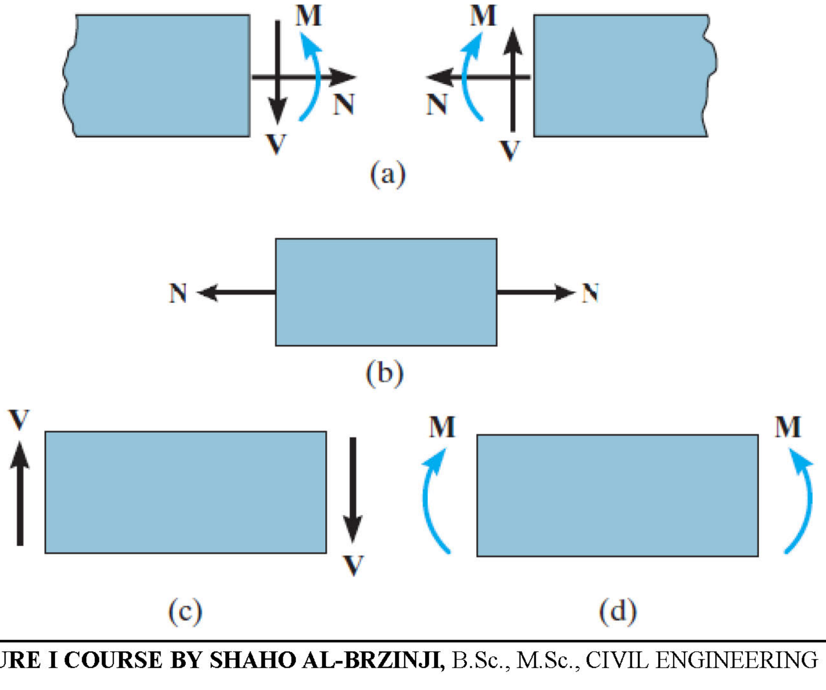
3 INTERNAL LOADINGS DEVELOPED IN STRUCTURAL MEMBERS

3.1 Internal Loadings at a Specified Point:

The internal load at a specified point in a member can be determined by using the *method of sections*. In general, this loading for a coplanar structure will consist of a normal force **N**, shear force **V**, and bending moment **M**, (Three-dimensional frameworks can also be subjected to a torsional moment, which tends to twist the member about its axis). It should be realized, however, that these loadings actually represent the *resultants* of the *stress distribution* acting over the member's cross-sectional area at the cut section. Once the resultant internal loadings are known, the magnitude of the stress can be determined provided an assumed distribution of stress over the cross-sectional area is specified.

3.2 Sign Convention:

Before presenting a method for finding the internal normal force, shear force, and bending moment, we will need to establish a sign convention to define their "positive" and "negative" values. Although the choice is arbitrary, the sign convention to be adopted here has been widely accepted in structural engineering practice, and is illustrated in **Fig.*a***. On the ***left-hand face*** of the cut member the normal force **N** acts to the **right**, the internal shear force **V** acts **downward**, and the moment **M** acts **counterclockwise**. In accordance with Newton's third law, an equal but opposite normal force, shear force, and bending moment must act on the ***right-hand face*** of the member at the section. Perhaps an easy way to remember this sign convention is to isolate a small segment of the member and note that ***positive normal force tends to elongate the segment,* Fig.*b*; *positive shear tends to rotate the segment clockwise,* Fig. *c* ; and *positive bending moment tends to bend the segment concave upward* Fig. *d***.



3.3 Procedure for Analysis:

Support Reactions

✓ Before the member is "cut" or sectioned, it may be necessary to determine the member's support reactions so that the equilibrium equations are used only to solve for the internal loadings when the member is sectioned.

Free-Body Diagram

✓ Keep all distributed loadings, couple moments, and forces acting on the member in their *exact location,* then pass an imaginary section through the member, perpendicular to its axis at the point where the internal loading is to be determined.

✓ After the section is made, draw a free-body diagram of the segment that has the least number of loads on it. At the section indicate the unknown resultants **N**, **V**, and **M** acting in their *positive* directions.

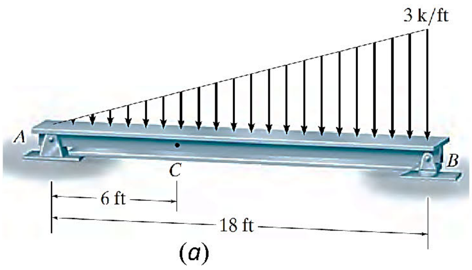
Equations of Equilibrium

✓ Moments should be summed at the section about axes that pass through the *centroid* of the member's cross-sectional area, in order to eliminate the unknowns **N** and **V** and thereby obtain a direct solution for **M**.

✓ If the solution of the equilibrium equations yields a quantity having a negative magnitude, the assumed directional sense of the quantity is opposite to that shown on the free-body diagram.

EXAMPLE 3.3.1

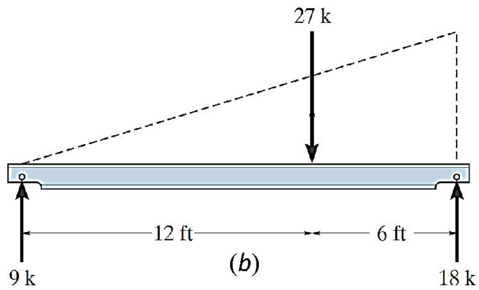
Determine the internal shear and moment acting at a section passing through point *C* in the beam shown in fig. *a*



(*a*)

Solution

Support Reactions. Replacing the distributed load by its resultant force and computing the reactions yields the results shown in fig *b*.



(*b*)

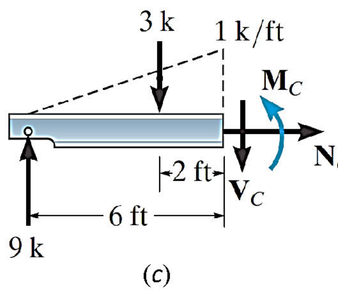
Free-Body Diagram. Segment *AC* will be considered since it yields the simplest solution, the figure. The distributed load intensity at *C* is computed by proportion, that is,

$$\frac{w_c}{6\text{ ft}} = \frac{3\text{ k/ft}}{18\text{ ft}} \Rightarrow w_c = 6\text{ ft} \times \frac{3\text{ k/ft}}{18\text{ ft}} = 1\text{ k/ft}$$

Equations of Equilibrium.

$$+\uparrow \sum F_y = 0; \quad 9 - 3 - V_C = 0 \qquad V_C = 6\text{ k}$$

$$+\circlearrowleft \sum M_C = 0; \quad -9(6) - 3(2) + M_C = 0 \qquad M_C = 48\text{ k.ft}$$

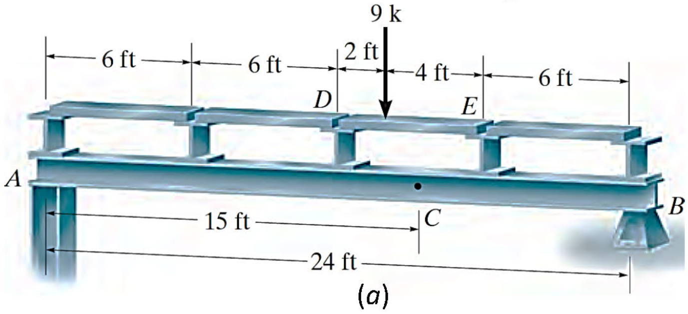


(*c*)

Note: This problem illustrates the importance of *keeping* the distributed loading on the beam until *after* the beam is sectioned. If the beam in **Fig.** ***b*** were sectioned at *C*, the effect of the distributed load on segment *AC* would not be recognized, and the result V_C **= 9 k** and M_C **= 54 k . ft** **would be wrong.**

EXAMPLE 3.3.2

The **9-k** force in Fig. *a* is supported by the floor panel ***DE***, which in turn is simply supported at its ends by floor beams. These beams transmit their loads to the simply supported girder ***AB***. Determine the internal shear and moment acting at point *C* in the girder.



(*a*)

Solution

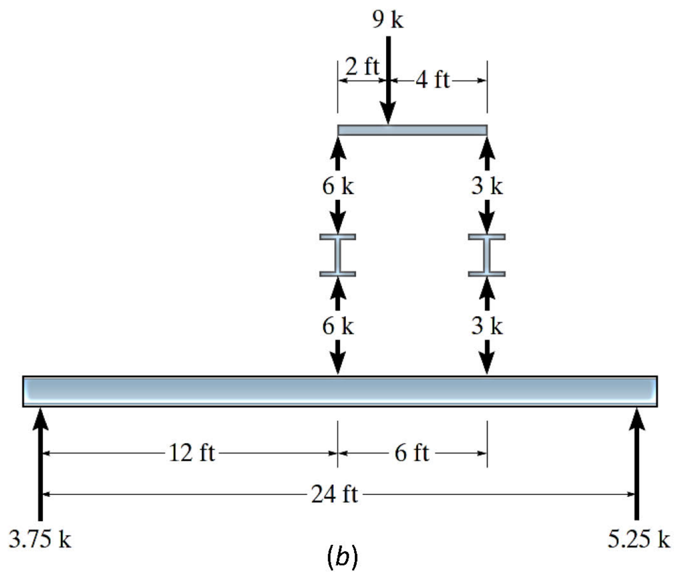
Support Reactions.

Equilibrium of the floor panel, floor beams, and girder is shown in **Fig.*b***. It is advisable to check these results.

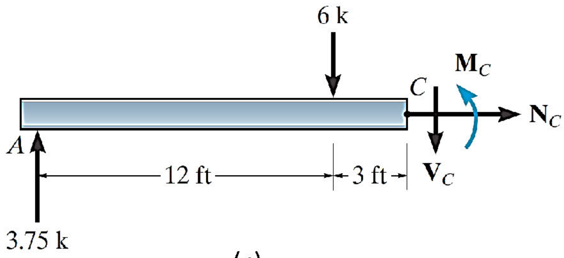
Free-Body Diagram.

The free-body diagram of segment *AC* of the girder will be used since it leads to the simplest solution, **Fig. *c***.

Note that there are *no loads* on the floor beams supported by *AC*.



(*b*)



(*c*)

Equations of Equilibrium.

$$+\uparrow \sum F_y = 0; \quad 3.75 - 6 - V_C = 0 \qquad V_C = -2.25 \text{ k}$$

$$+\circlearrowleft \sum M_C = 0; \quad -3.75(15) - 6(3) + M_C = 0 \qquad M_C = 38.25 \text{ k.ft}$$

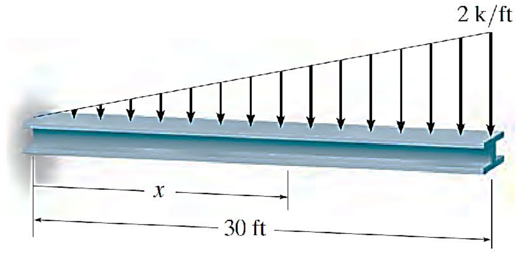
3.4 Shear and Moment Functions

The design of a beam requires a detailed knowledge of the variations of the internal shear force *V* and moment *M* acting at each point along the axis of the beam.
The internal normal force is generally not considered for two reasons:

1. In most cases the loads applied to a beam act perpendicular to the beam's axis and hence produce only an internal shear force and bending moment.
2. For design purposes the beam's resistance to shear, and particularly to bending, is more important than its ability to resist normal force.

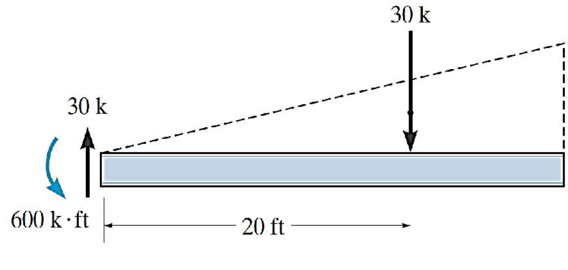
The variations of *V* and *M* as a function of the position *x* of an arbitrary point along the beam's axis can be obtained by using the method of sections. It is necessary to locate the imaginary section or cut at an arbitrary distance *x* from one end of the beam rather than at a specific point.
In general, the internal shear and moment functions will be discontinuous, or their slope will be discontinuous, at points where the type or magnitude of the distributed load changes or where concentrated forces or couple moments are applied. Because of this, shear and moment functions must be determined for each region of the beam located *between* any two discontinuities of loading.

EXAMPLE 3.4.1
Determine the shear and moment in the beam shown in **Fig. *a*** as a function of ***x***.



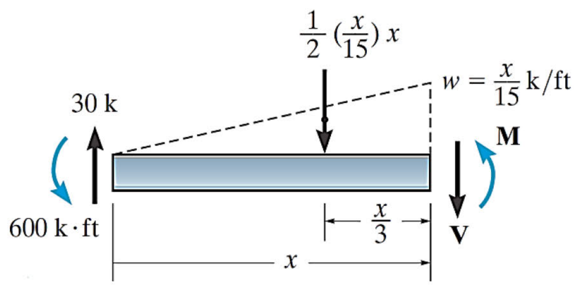
(a)

Solution
Support Reactions. For the purpose of computing the support reactions, the distributed load is replaced by its resultant force of **30 k, Fig. *b***. It is important to remember, however, that this resultant is not the actual load on the beam.



(b)

Shear and Moment Functions. A free-body diagram of the beam segment of length x is shown in **Fig. c**. Note that the intensity of the triangular load at the section is found by proportion; that is, ***w*/*x* = 2/30, *w* = *x*/15.**
With the load intensity known, the resultant of the distributed loading is found in the usual manner as shown in the figure. Thus,



$$+\uparrow \sum F_y = 0; \quad 30 - \frac{1}{2}\left(\frac{x}{15}\right)x - V = 0$$

$$\mathbf{V = 30 - 0.0333x^2}$$

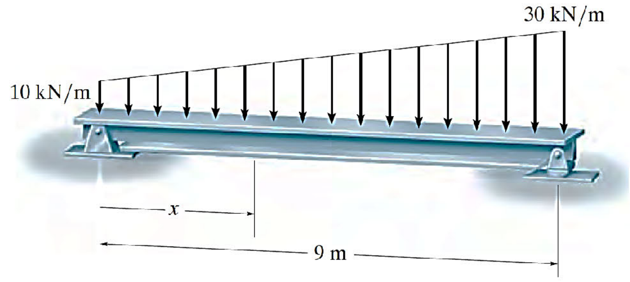
$$+\circlearrowleft \sum M_{Section} = 0; \quad 600 - 30x + \left[\frac{1}{2}\left(\frac{x}{15}\right)x\right]\frac{x}{3} + M = 0$$

$$\mathbf{M = -600 + 30x - 0.0111x^3}$$

Note that ***dM*/*dx* = *V*** and ***dV*/*dx* = -*x*/15 = *w*** which serves as a check of the results.

EXAMPLE 3.4.2

Determine the sh ear and moment in the beam shown in **Fig.*a*** as a function of ***x***.

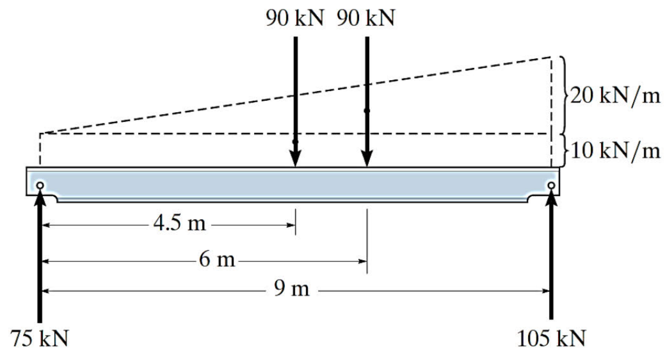


(a)

Solution

Support Reactions.

To determine the support reactions, the distributed load is divided into a triangular and rectangular loading, and these loadings are then replaced by their resultant forces. These reactions have been computed and are shown on the beam's free body diagram, **Fig.*b***.



(b)

Shear and Moment Functions.

A free-body diagram of the cut section is shown in **Fig.*c***. As above, the trapezoidal loading is replaced by rectangular and triangular distributions. Note that the intensity of the triangular load at the cut is found by proportion. The
resultant force of each distributed loading and its location are indicated. Applying the equilibrium equations, we have

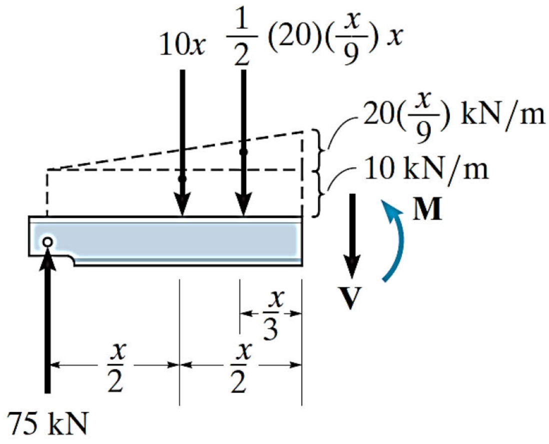
$$+\uparrow \sum F_y = 0; \quad 75 - 10x - \left[\frac{1}{2}(20)\left(\frac{x}{9}\right)x\right] - V = 0$$

$$\boldsymbol{V = 75 - 10x - 1.11x^2}$$

$$+\circlearrowleft \sum M_{Section} = 0;$$

$$-75x + (10x)\left(\frac{x}{2}\right) + \left[\frac{1}{2}(20)\left(\frac{x}{9}\right)x\right]\frac{x}{3} + M = 0$$

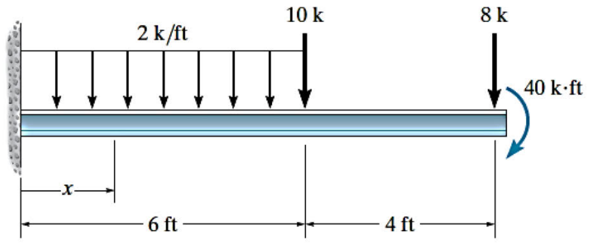
$$\boldsymbol{M = 75x - 5x^2 - 0.370x^3}$$



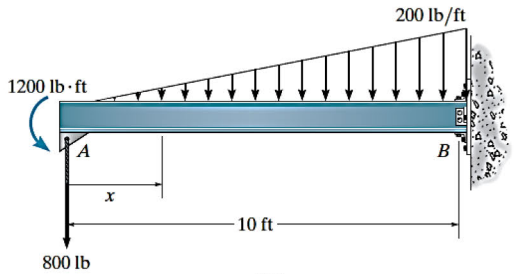
(c)

Hw.1

Determine the shear and moment throughout the beams as functions of x. And draw the shear and moment diagrams



(1)



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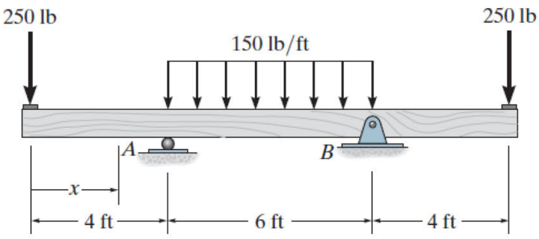

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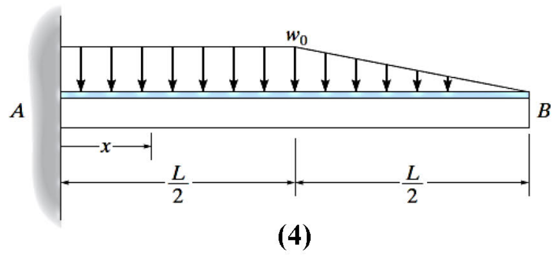

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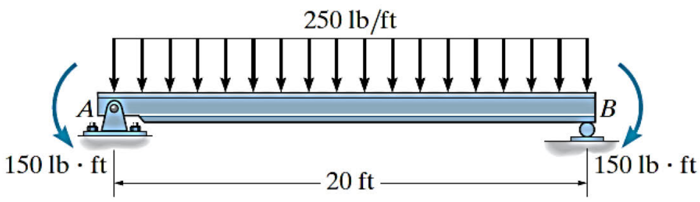

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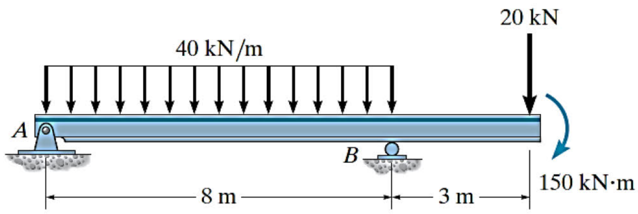

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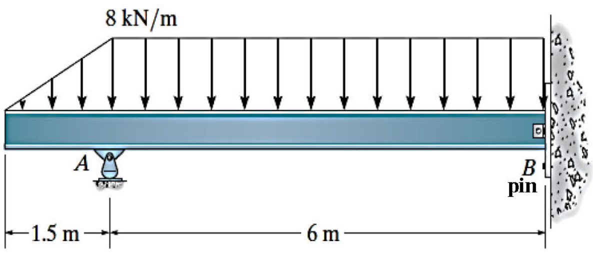

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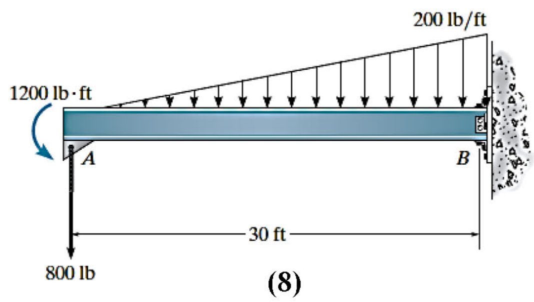

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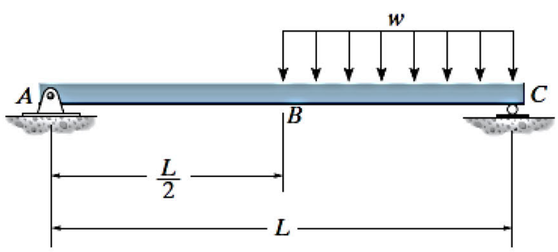

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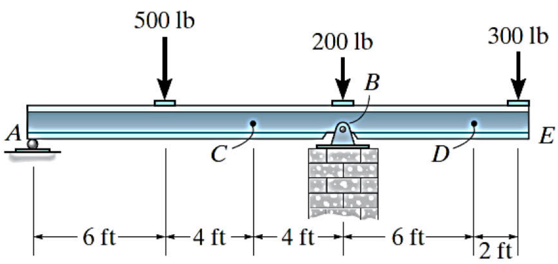
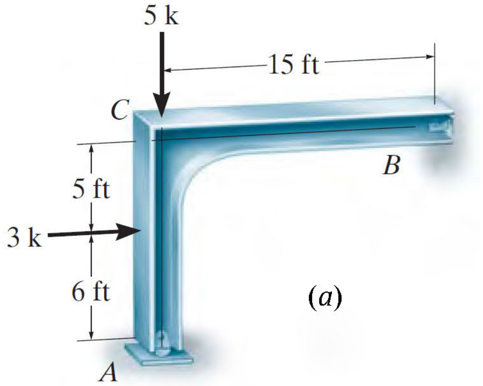


(10)

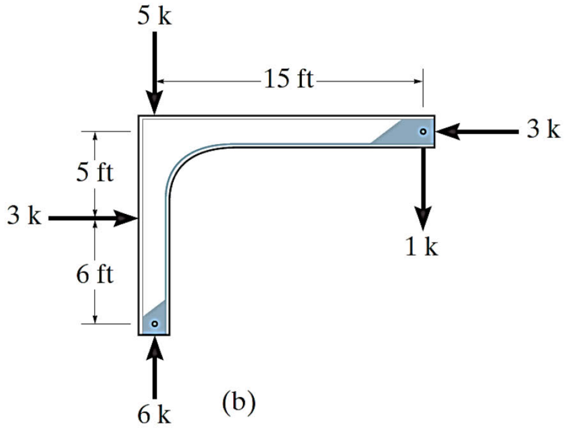
3.5 Shear and Moment Diagrams for a Frame

Frame is composed of several connected members that are either fixed or pin connected at their ends. The design of these structures often requires drawing the shear and moment diagrams for each of the members.
To analyze any problem first determining the reactions at the frame supports. Then, using the method of sections, we find the axial force, shear force, and moment acting at the ends of each member.

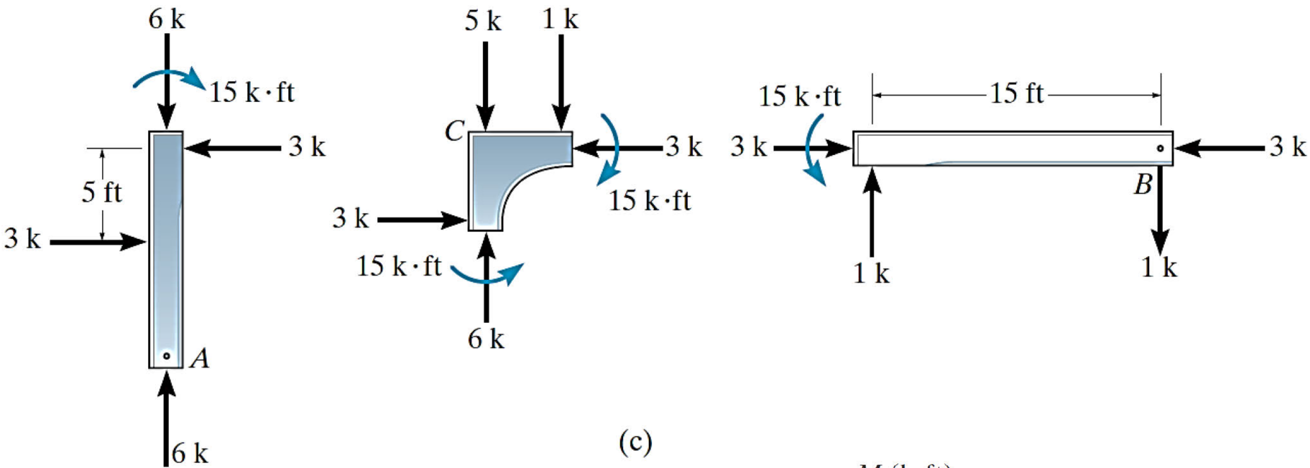
EXAMPLE 3.5.1
Draw the moment diagram for the tapered frame shown in **Fig. *a***. Assume the support at ***A*** is a roller and ***B*** is a pin.



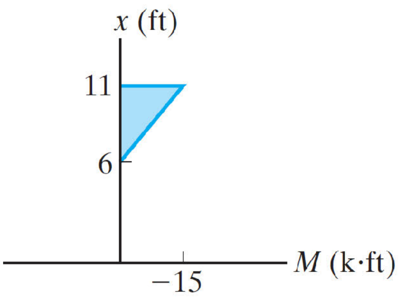
(*a*)



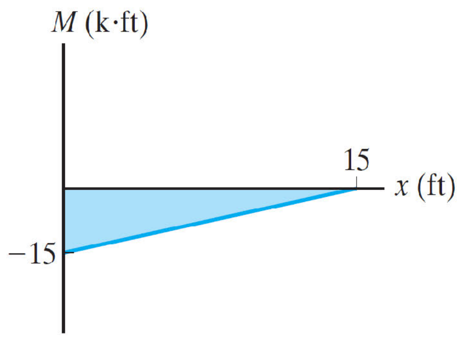
(b)



(c)



member *AC*



member *CB*

EXAMPLE 3.5.2

Draw the shear and moment diagrams for the frame shown in **Fig. *a***. Assume A is a pin, C is a roller, and B is a fixed joint. Neglect the thickness of the members.

Solution

Notice that the distributed load acts over a length of

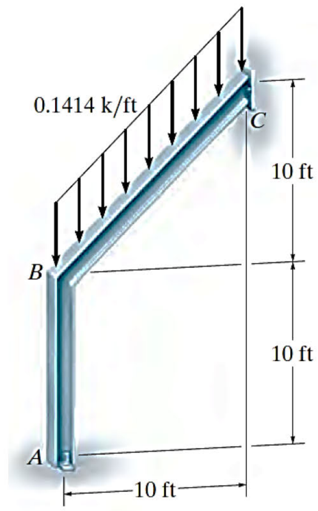
$$10 \text{ ft} \sqrt{2} = 14.14 \text{ ft}$$

The reactions on the entire frame are calculated and shown on its free-body diagram, **Fig. *b***. From this diagram the free-body diagrams of each member are drawn, **Fig. *c***.

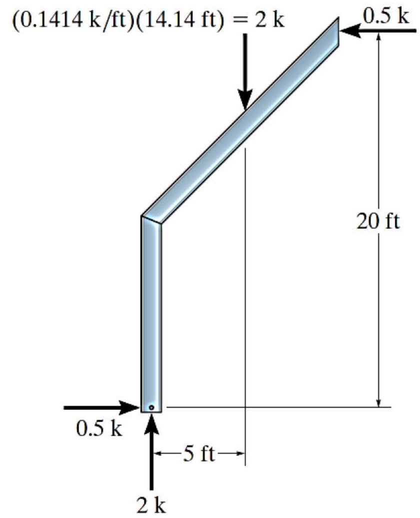
The distributed loading on BC has components along BC and perpendicular to its axis of

$$(0.1414 \text{ k/ft}) \cos 45° = (0.1414 \text{ k/ft}) \sin 45° = 0.1 \text{ k/ft}$$

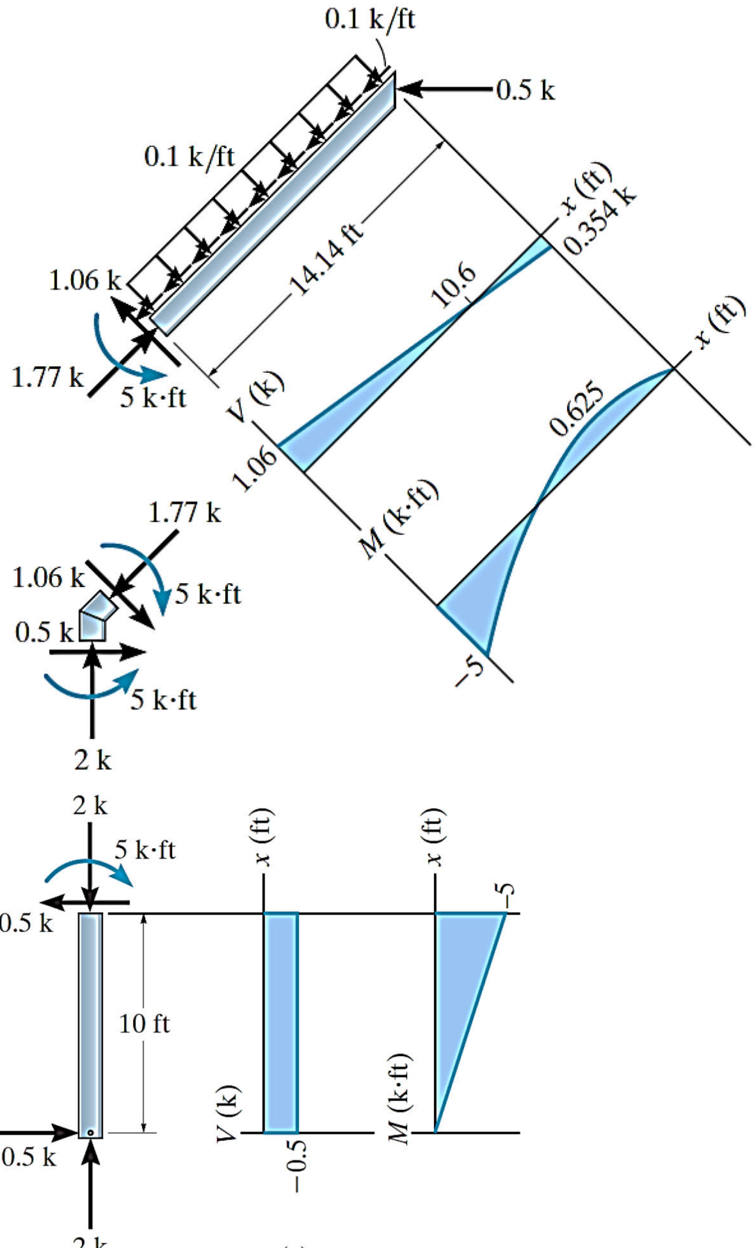
the shear and moment diagrams are also shown in **Fig. *c***.



(a)



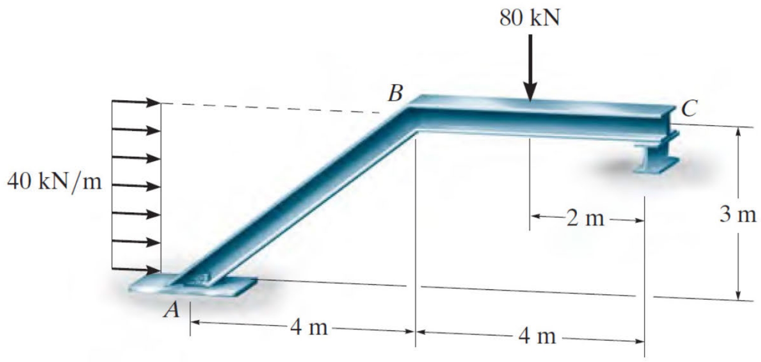
(b)



(c)

EXAMPLE 3.5.3

Draw the shear and moment diagrams for the frame shown in ***Fig. a***. Assume ***A*** is a pin, ***C*** is a roller, and ***B*** is a fixed joint.

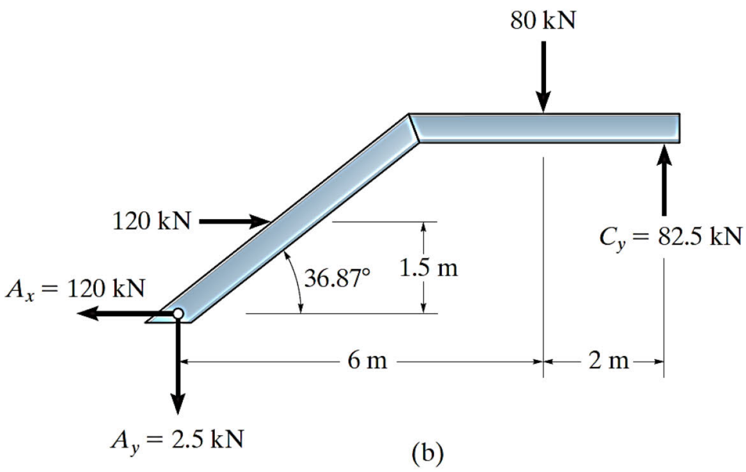


(a)

Solution

Support Reactions.

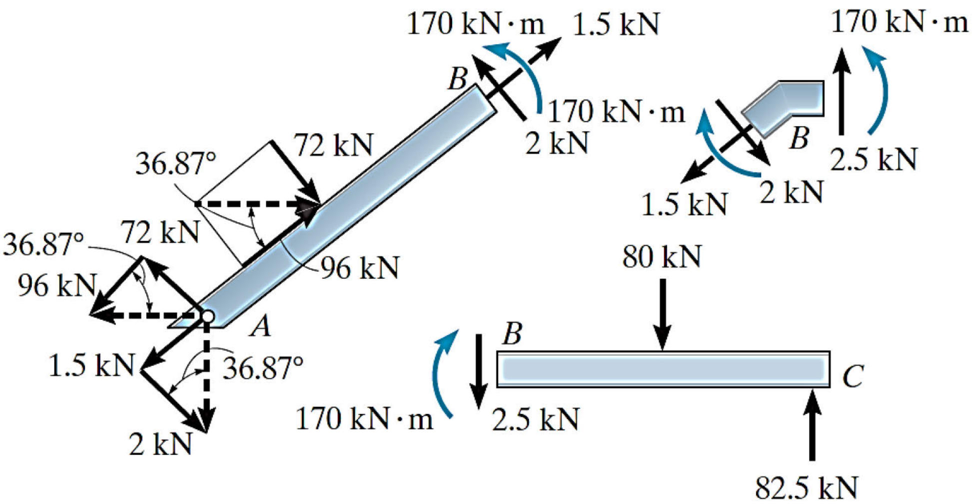
The free-body diagram of the entire frame is shown in **Fig.*b***. Here the distributed load, which represents wind loading, has been replaced by its resultant, and the reactions have been computed. The frame is then sectioned at joint ***B*** and the internal loadings at ***B*** are determined, **Fig.*c***. As a check, equilibrium is satisfied at joint ***B***, which is also shown in the figure.



(b)

Shear and Moment Diagrams.

The components of the distributed load, **(72 kN)/(5 m) = 14.4 kN/m** and **(96 kN)/(5 m) = 19.2 kN/m**, are shown on member ***AB*, Fig. *d***. The associated shear and moment diagrams are drawn for each member as shown in **Figs. *d*** and ***e***.



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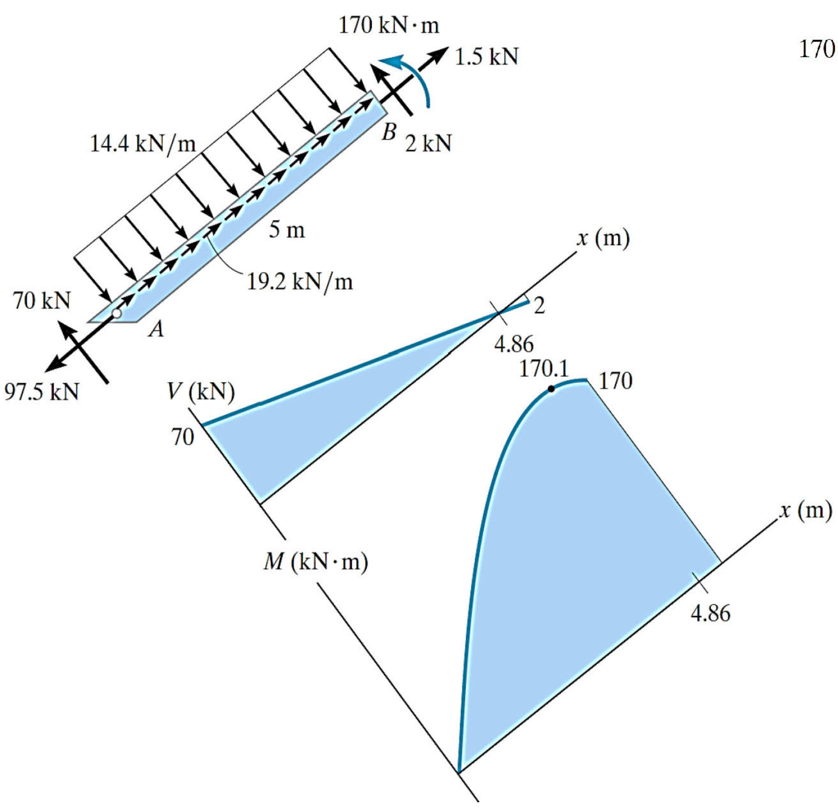
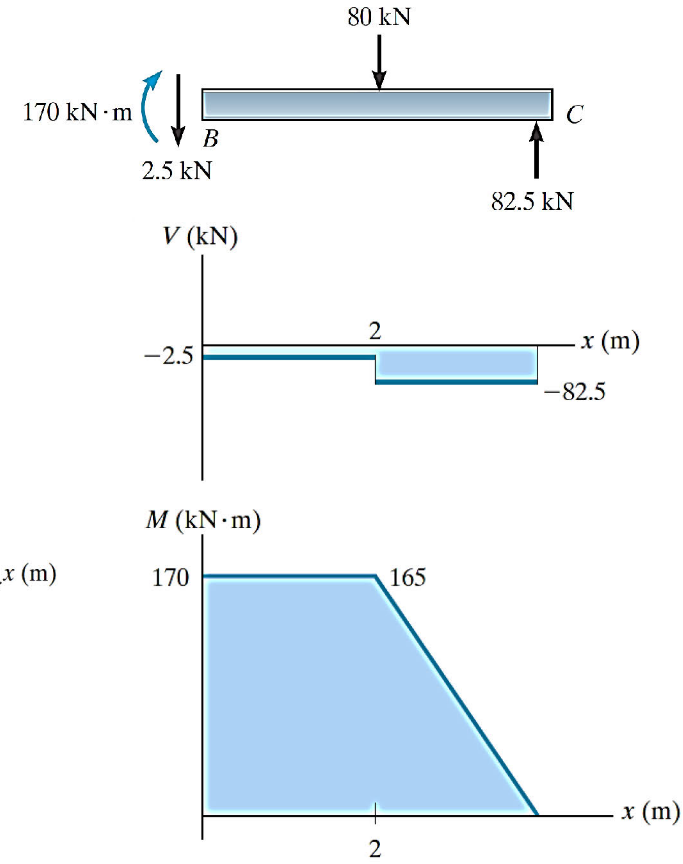
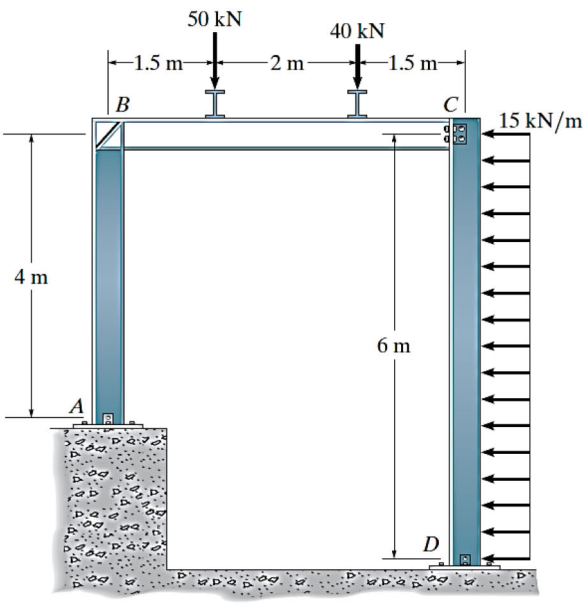


(d)

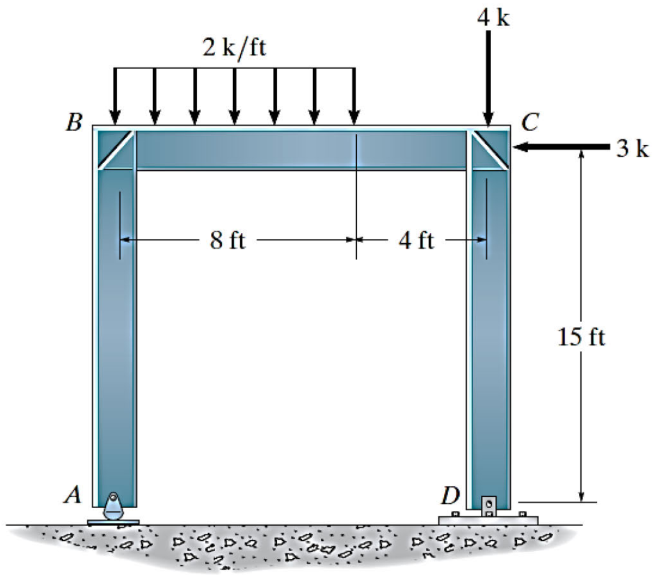


(e)

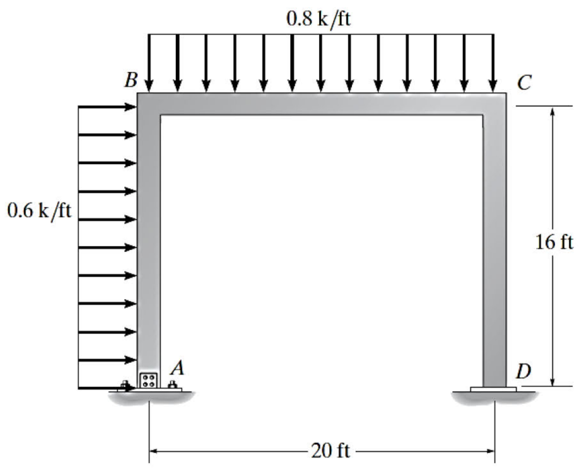
Hw.2 Draw the shear and moment diagrams for each of the three members of the frame. Assume the frame is pin connected at ***A***, ***C***, and ***D*** and there is a fixed joint at ***B***.



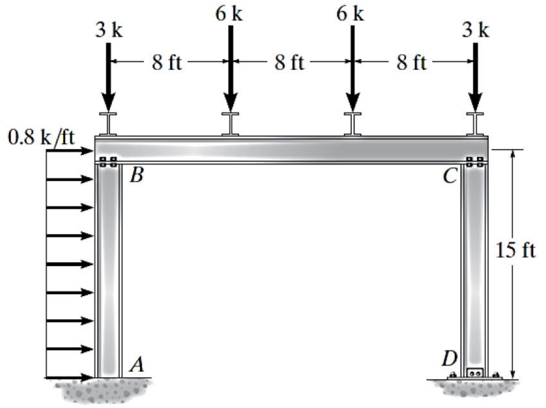
Hw.3 Draw the shear and moment diagrams for each member of the frame. Assume ***A*** is a rocker, and ***D*** is pinned.



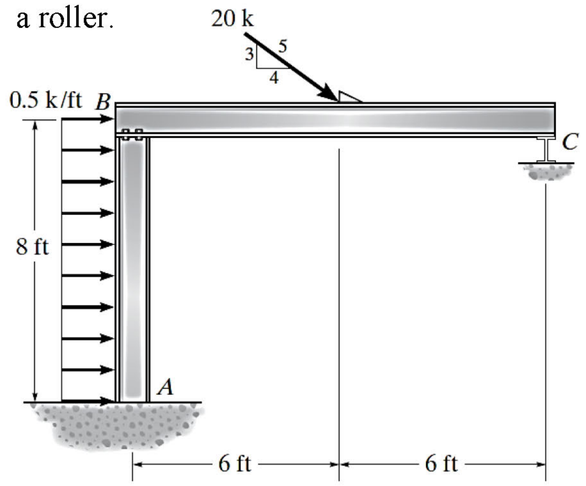
Hw.4 Draw the shear and moment diagrams for each member of the frame. Assume the support at ***A*** is a pin and ***D*** is a roller.



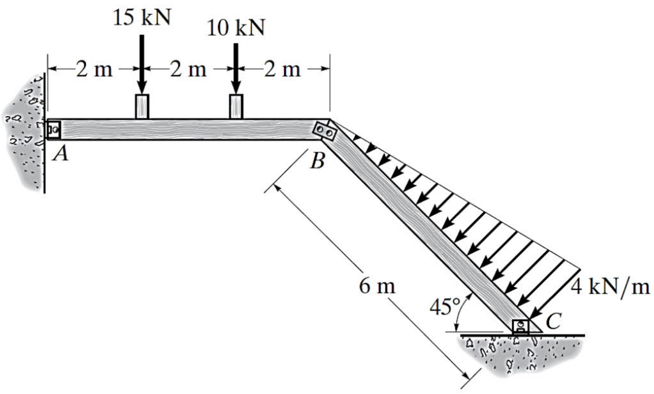
Hw.5 Draw the shear and moment diagrams for each member of the frame. Assume the frame is pin connected at ***B***, ***C***, and ***D*** and ***A*** is fixed.



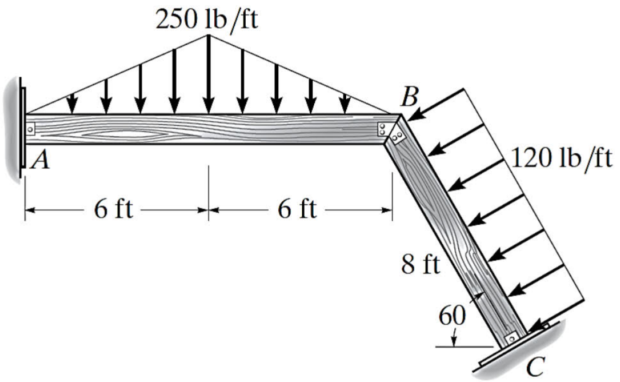
Hw.6 Draw the shear and moment diagrams for each member of the frame. Assume ***A*** is fixed, the joint at ***B*** is a pin, and support ***C*** is a roller.



Hw.7 Draw the shear and moment diagrams for each member of the frame. The members are pin connected at ***A***, ***B***, and ***C***.



Hw.8 Draw the shear and moment diagrams for each member of the frame. The joints at ***A***, ***B***, and ***C*** are pin connected.



Hw.9 Draw the shear and moment diagrams for each member of the frame.